

HARVESTING POLICIES AND THE FISHERY CONSERVATION
AND MANAGEMENT ACT OF 1976

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The Fishery Conservation and Management Act of 1976 (FCMA), or Public Law 94-265, was passed by Congress to prevent future over-exploitation and overcapitalization in the fishing industry, as well as to develop underutilized fisheries to help meet the expected future increase in demand for fish products. The FCMA creates eight regional fishery management councils (hereafter just called the councils) to develop management plans for the fisheries within their jurisdiction. The councils are required to determine the "optimum yield" (OY), to allocate the yield between foreign and domestic users, and to suballocate within these groups.

As described in the FCMA, OY implies management for a variety of objectives. A management plan must include biological, recreational, economic, and social goals in determining OY and how the yield is allocated. For most fisheries, the optimum levels for all the objective considered separately are in conflict, so that one task of the councils is to develop procedures to find an optimum among multiple, conflicting objectives. In practice, this may mean developing several complementary techniques. A first step might be to find the utility functions for each of the interest groups, a procedure demonstrated by Keeney (1977) for managing salmon on the Skeena River. Then, given the resulting, conflicting utility functions, a second step might be to calculate the set of "efficient" or "Pareto optimal" policies, in order to reduce the number of policies the council need consider. Finally a bargaining procedure or other interactive method might be developed for the council to decide on one final plan.

Research in several of these areas is being conducted by the Honolulu Laboratory, National Marine Fisheries Service, and by the Western Pacific Regional Fishery Management Council. In this paper I present results for the second step of the process described above. The set of efficient or Pareto optimal policies is described for a class of dynamic, stochastic harvesting models relevant to fisheries management. Further, an extension of the usual dynamic programming formulation that allows a vector-valued return function rather than the usual real value return function.

I have shown previously (Mendelsohn 1976a, 1976b, 1977; Mendelsohn and Sobel 1977) that dynamic, stochastic harvesting models are highly structured capital accumulation models that possess a highly structured optimal policy function. Similar structure in the vector maximization problem makes it possible to describe the set of efficient policies solely from knowing the single objective optimal policy functions. Also, I show that it is possible to obtain readily several points of interest within the efficient set.

Let the k -vector J^* be the optimum optimum vector, and let the k -vector J be the expected return from a given policy. That is, J^{*i} is the expected return for objective i when it is the sole objective and an optimal policy is followed. J^i is the expected return to objective i following some (any) policy.

The "p-regret function" $R_p(J)$ is defined as:

$$R_p(J) = \left(\sum_{i=1}^k (J^{*i} - J^i)^p \right)^{1/p} \quad p < \infty$$

where it is convenient to let $R_{\infty}(J) \equiv \max_i (J_i^* - J_i)$. It is desired

to find the feasible return vector J that minimizes $R_p(J)$ for different values of p . I consider the case $p = 1$, or total group regret, and $p = \infty$, the Chebyshev problem. A third criterion of interest is the policy that is optimal for a convex combination of the k -objective functions using $\frac{1}{k}$ for the weights. I refer to this as the "equitable" policy. The dynamic, stochastic models considered in this paper have the interesting property that a policy which is equitable also is Chebyshev optimal and minimizes the total regret.

II. Notation and Development

Describing a vector-valued stochastic model requires a great deal of notation. The glossary at the end contains a complete list of the notation used. As much as possible the notation is consistent with Mendelssohn (1976b) or Mendelssohn and Sobel (1977), since the results presented here rely on the single-objective methods in those papers.

The fishery is being managed for a planning horizon of T periods, $T < \infty$. The periods are subscripted by t $1 \leq t \leq T$. Alternately, $n = T - t + 1$, the number of periods remaining in the planning, is used as the subscript. The infinite horizon problem can be treated by allowing n to approach infinity. At the beginning of each period an initial stock size x_t is observed, with $x \in X$, the set of possible stock sizes. The stock is harvested by j user groups, and each group catches z_t^j , $j = 1, \dots, j$ during period t . Let the j -vector $z_t = (z_t^1, \dots, z_t^j)$ be the

vector of the j catches during period t . The stock size in period $t + 1$ is assumed to be a random function $s[\cdot, \cdot]$ of the stock left after harvesting has ceased, that is:

$$x_{t+1} = s \left[x_t - \sum_{i=1}^j z_t^i, D_t \right] \quad (2.1)$$

where D_1, D_2, \dots, D_T are independent, identically distributed random variables distributed as the generic random variable D .

For a given stock size and harvest vector (x, z) , each of the j users has a one period return given by the real-valued function $g_t^1(x, z)$. There may be $k - j \geq 0$ other groups that have an interest in how the stock is managed. Their one period returns are denoted by $g_t^i(x, z)$, $i = j + 1, \dots, k$. Let the k -vector $G_t(x, z) = (g_t^1(x, z), \dots, g_t^k(x, z))$. The natural constraints on the problem are:

$$\begin{aligned} x_t &\geq \sum_{i=1}^j z_t^i \\ x_t, z_t^i &\geq 0 \quad i = 1, \dots, j \quad \text{for all } t \end{aligned} \quad (2.2)$$

For a k -vector valued function G with components g^i , $i = 1, \dots, k$ a policy z^* (resp. a return vector J^*) is termed "efficient" or undominated if there is no other feasible policy (resp. no other feasible return J) satisfying:

$$G(x, z) \geq G(x, z^*) \underline{1/} \quad (2.3)$$

That is, a policy is efficient if there is no other feasible policy that does at least as well in $k-1$ of the objectives and strictly better in the k^{th} objective.

For any k -vector valued function $G(x, z)$, let X be the set of all possible states, $Z(x)$ the set of feasible decisions at $x \in X$, and $C \equiv \{(x, z) : x \in X; z \in Z(x)\}$. Let Λ be the set of efficient return vectors of G for $(x, z) \in C$, and Ω the set of G such that $(x, z) \in C$. Define the operator $vmax$ as a map from $\Omega \rightarrow \Lambda$, that is it maps the set of feasible vector returns into the set of efficient vector returns. Any $G(x, z) \in \Lambda$ implies a subset of C such that (x, z) is an element of the subset if and only if $G(x, z) \in \Lambda$. At times I use the slightly abused notation of having $vmax$ map C into its efficient subset. The use, however, should be clear from the context.

For the dynamic problem, let E denote the expectation operator. Then the problem of finding the set of efficient policies can be expressed as:

$$vmax \ E \left(\sum_{t=1}^T G_t(x_t, z_t) \right) \quad (2.4)$$

subject to (2.1), (2.2)

1/ When comparing two k -vectors x and y , $x \geq y$ implies $x^i \geq y^i$ for all i ; $x \geq y$ implies $x \geq y$ but $x \neq y$; $x > y$ implies $x^i > y^i$ for all i .

A history H_t is a feasible sequence of states, decisions, and random variables up to time t , that is:

$$H_t = \left\{ x_1, z_1, D_1; x_2, z_2, D_2; \dots; x_{t-1}, z_{t-1}, D_{t-1}; x_t \right\}$$

By a policy, I mean a function which for each period and all possible histories describes what decision to make. Let $V_t(\delta|H_t)$ be the expected value of the k objectives when history H_t has been observed, and the possibly nonstationary policy δ is followed for the remainder of the planning horizon.

Theorem 1 is the central mathematical theorem in this paper. Results along the same lines certainly are implicit in Sobel (1975, 1977); however, I know of no other reference where the dynamic programming problem is expanded to treat the vector maximum problem exactly as it is here. The proof, and the proofs of all results in this paper, can be found in the Appendix.

Theorem 2.1 For problem (2.4)

- i) A policy is efficient from period 1 if it is efficient from period t onwards, for all $t \leq T$.
- ii) An efficient policy only depends on the past history H_t through the present state x_t ; that is, the expected value of a possibly nonstationary policy δ given H_t satisfies:

$$V_t(\delta|H_t) = V_t(\delta|x_t)$$

for all H_t , δ , and t .

□

Corollary 2.1 Let $f_n^{\text{eff}}(x)$ be defined as equal to $V_n(\delta|x)$ for some (any) efficient policy δ . Then z is an efficient decision for state x in period n if and only if it is undominated for the k -vector valued function given by:

$$G_n(x, z) + E f_{n-1}^{\text{eff}}(s[x - \sum_i z^i, d^i])$$

□

If we modify f_n^{eff} to be the set of efficient vector returns, and have $G + E f_{n-1}^{\text{eff}}$ denote the set of values given by the value of G plus the value of each element in the set $E f_{n-1}^{\text{eff}}$, then corollary 2.1 leads to the multiobjective extension of dynamic programming, which can be formulated as:

$$f_0^{\text{eff}}(\cdot) \equiv 0$$

$$f_n^{\text{eff}}(x) = \text{vmax} \left\{ G_n(x, z) + E f_{n-1}^{\text{eff}}(s[x - \sum_i z^i, d]) \right\}$$

To describe the set of dynamic efficient policies, it becomes only necessary to describe the set of efficient actions in period n , given the state x . This point to set mapping is the equivalent of the usual optimal policy function in single objective dynamic programming. Given that a decision z is efficient in period n , the set of resulting efficient policies is to choose any efficient action in the following periods.

A closely related problem is:

$$\begin{aligned} \text{maximize } EJ &= E \sum_{t=1}^T \sum_{i=1}^k \lambda^i g_t^i(s_t, z_t) \\ \text{subject to } \sum_{i=1}^k \lambda^i &= 1, 0 \leq \lambda^i \leq 1; (2.1), (2.2) \end{aligned} \quad (2.5)$$

From standard dynamic programming arguments, a solution to (2.5) satisfies the system of recursive equations:

$$f_0^\lambda(\cdot) \equiv 0 \quad (2.6)$$

$$f_n^\lambda(x) = \max_{0 \leq \sum z^i \leq x} \{J_n^\lambda(x, z)\}$$

$$\text{where } J_n^\lambda(x, z) = \left\{ \sum_{i=1}^k \lambda^i g^i(x, z) + E f_{n-1}^\lambda \left(s[x - \sum z^i, d] \right) \right\}$$

If each of the g^i are concave, equation (2.6) and corollary 2.1 imply that there exist efficient policies which change the weighting factors λ^i between periods. At first glance policies of this form may appear to be of little interest. However, the desirability of policies that have changing implied weightings can occur for several reasons. First, given the actual realization of the random process, the actual returns may be very different from the expected returns. Certain objectives may be more sensitive to this variation, and changing to a different implied weighting may compensate for this.

Second, experience may teach us that what we thought we preferred is not what we actually preferred. That dynamic efficient policies exist that change the weightings with time allows us to change preferences as we learn about the system. Rather than being uninteresting, corollary 2.1 tells us that we can switch the implied weighting midstream and still have an efficient policy for the whole planning horizon. This flexibility seems desirable.

Finally, for a real function of one-argument, f' denotes the derivative of $(f: R \rightarrow R)$ and ∇f denotes the gradient vector. For a function of two arguments $f^{[1]}$ denotes the gradient with respect to the first vector, and $f^{[2]}$ the gradient with respect to the second vector. $\nabla^i f$, $f^{[1]i}$, $f^{[2]i}$ denote the i th component of the vector. At times, one-sided or directional derivatives are appropriate; however, I feel these are clear from the context.

III. Result

A. Single decision variable

In some fisheries, only one of the groups interested in the stock actually harvest it. For example, the northern anchovy (Engraulis mordax Girard) is exploited by a reduction fishery, which uses the anchovy mainly for meal. However, the northern anchovy is considered an important food source for several important sport fish. Recreational fishing associations have expressed a strong interest in how the fishery is managed. The government has its own goals--both to insure the health of the stock and of the industry given a population that exhibits large natural fluctuations.

I derive results for the bicriterion case, and then show that similar results are valid for $k > 2$. I assume hereafter (unless otherwise stated) that $s_t[\cdot, \cdot]$ is a stationary function, and that G_t depends on time only through a discount factor α , $0 < \alpha \leq 1$. That is

$$G_t(x, z) = \alpha^{t-1} G(x, z)$$

I assume that the one period return (or utility) for each interest group depends only on the allowable catch, and that the following assumptions on g^i are valid for each user i :

- (i) $g^i(\cdot)$ is concave and continuous on the set $C = \{(x, z) : x \in X; 0 \leq z \leq x\}$
- (ii) $g^i(\cdot)$ is nondecreasing; $g^i(\cdot) < \infty$
- (iii) $g^i(\cdot)$ is uniformly bounded from below

and I assume for the transition function $s[\cdot, \cdot]$ that:

- (iv) $s[\cdot, d]$ is concave and continuous for each fixed value d on the set

$$Y = U$$

Assumptions (i)-(iv) are discussed in greater detail in Mendelssohn (1976b) or Mendelssohn and Sobel (1977). If the demand curve is such that per unit price (benefit) is nonincreasing with total supply, then $p(z) \cdot z$, the total value of the harvest satisfies assumptions (i)-(iii). Also, if the preference structure of a decision-maker is additive and risk-adverse, or multiplicative such that the log of the utility is risk-adverse, then the utility function determined by that preference structure satisfies (i)-(iii) (see Keeney 1974 for a complete discussion of conditions that generate utilities with these forms).

Assumption (iv) is satisfied by many production models. I believe that if $s[x - z, d] = ds[x - z]$ then it is sufficient that $s[\cdot]$ be pseudoconcave, which would include a more general class of production models. However, except for the linear returns case, I have not been able to prove this.

As described, the problem does not include allocation, but rather involves groups with differing valuations of the work of an allowed catch in any period. Often problems stated in somewhat different terms from my assumptions can be put into this form. For example, the Sport Fishing Institute (SFI) has called for a based stock policy to manage the northern anchovy. They prefer that the industry catch be set at 10% of the surplus over a fixed size spawning population, or else zero. It follows from the single objective results in Mendelssohn (1976b) or Mendelssohn and Sobel (1977) that this policy is optimal for some return function satisfying assumptions (i)-(iii). Assumption (iv) can be found in MacCall (1976) or Mandell (1975). Other conservation aimed objectives can often be put into a form satisfying assumptions (i)-(iii) by carefully choosing the objective function.

Theorem 3.1 is the main result for this model. It states that knowledge of the single-objective optimal policy functions is sufficient to calculate the set of Pareto optimal policies. Let $z_n^{1*}(x)(z_n^{2*}(x))$ be the optimal policy functions when objective one (two) is the sole objective being considered.

Theorem 3.1 For problem (2.4), let $k = 2$, $j = 1$. For fixed x , assume $z_n^{*1}(x)$, $z_n^{*2}(x)$ are known for the two single objective problems, with $z_n^{*1}(x) \leq z_n^{*2}(x)$. Then:

- i) A decision z is an element of a Pareto optimal policy if and only if z is contained in the set $\{z \mid z_n^{*1}(x) \leq z \leq z_n^{*2}(x)\}$
- ii) The set of Pareto optimal policies for each n and $x \in X$ is to choose a z as in (i) and any Pareto optimal policy in period $n - 1$. □

A conclusion from theorem 3.1 is that the wider the disagreement among the users, the greater the number of Pareto optimal policies. This makes it all the more important to find a smaller number of policies that have other desirable properties. One such policy might be to choose

$z = 1/2(z_n^{*1}(x) + z_n^{*2}(x))$, the halfway point between the two desired policies. Other policies, as mentioned in section I, are ones that are equitable, Chebyshev optimal and minimize the total group regret.

Theorem 3.2 relates these policies.

Theorem 3.2 Under the assumption of theorem 3.1, there exists an equitable policy that also minimizes the total regret and minimizes the maximum regret. □

Only rarely will this policy be equivalent to choosing

$$z = 1/2(z_n^{1*}(x) + z_n^{2*}(x)).$$

These two policies certainly seem to be

reasonable starting points for any "bargaining" procedure to determine a final policy.

So far the results have been limited to two objectives. However, corollary 3.1(2) states that the results are equally valid for $k \geq 2$. Again, let $z_n^{i*}(x)$ be the optimal policy function when objective i is the only objective considered.

Corollary 3.1 The results of theorem 3.1, with $k \geq 2$

and $z_n^{1*}(x) = \text{minimum}_{1 \leq i \leq k} \{z_n^{i*}(x)\}$ and $z_n^{2*}(x) = \max\{z_n^{i*}(x)\}$ are valid,

and theorem 3.2 remains correct as stated.

□

It should be noted that in using the regret functions (or even weighting) I tacitly assumed all of the objective functions are on an equivalent scale. The results are valid if they are not, but the interpretation becomes clouded. Such metrics are always affected most by the largest return with the largest spread (greater rates of change) and without proper weighting will not have any intuitive notion of fairness.

B. Allocating catch among users

In the last section, there was only one user of the resource, and the conflict was over how large a catch should be allowed that user. A second common problem is where there are several users of the resource.

A policy must decide both total allowable catch and the allocation of that catch. The problem is a dynamic allocation model.

Consider first the two user, two objective case, that is $j = k = 2$. I assume that the one period return to each user depends only on the catch allotted to that user, and that $g^i(\cdot)$ satisfies assumption (i)-(iii) for each i , and that the transition function satisfies assumption (iv).

The set of Pareto optimal policies can be defined by knowing three relatively simple functions. First, I assume that the single objective optimal policy functions, $z_n^{1*}(x)$ and $z_n^{2*}(x)$ are known. Second, I assume that the curve (as a function of λ) where $\lambda g^{2'}(z_1) = (1-\lambda)g^{2'}(z_2)$ is known on the (z_1, z_2) coordinates. For each λ , the solution (z_1, z_2) to the equality is the point where the weighted price (derivative) to the one user just equals the weighted price to the other user, so that a small loss to the one is just offset by the gain to the other. If the problem was not dynamic, the curve defined by $\lambda g^{1'}(z_1) = (1-\lambda)g^{2'}(z_2)$ would clearly define all efficient solutions. It is appealing that the dynamic policy should also be related to this curve, as well as to the single objective optimal policy functions. Theorem 3.3 states that this is in fact the case.

Theorem 3.3 Assume in (2.4) that $j = k = 2$, that $g^i(z)$ depends on z only through z^i , that assumptions (i)-(iv) are valid, and that the two single objective policy functions $z_n^{1*}(x)$, $z_n^{2*}(x)$ are known. Then:

- i) For each n , and $x \in X$, there is a Pareto optimal policy for z such that z is contained in the set $\{z \mid z_n^{1*}(x) \leq z^1 + z^2 \leq z_n^{2*}(x)\}$ assuming $z_n^{1*}(x) \leq z_n^{2*}(x)$.
- ii) Each of these policies is equivalent to a point on the curve (as λ varies) where $\lambda g^{1'}(z^1) = (1 - \lambda)g^{2'}(z^2)$ on the interval $[z_n^{1*}(x), z_n^{2*}(x)]$ or else is at an end point of the interval.
- iii) A Pareto optimal policy is to choose z as in (i) and any Pareto optimal policy for the state in period $n - 1$.

□

As in the last section, the greater the conflict between the two single objective policies, the larger the set of Pareto optimal policies. Again, are there certain policies that can be selected from the set that are prime candidates? Some such policies are any Pareto policy such that $z^1 = z^2$. Of these, the one where $z^1 + z^2 = 1/2(z_n^{1*}(x) + z_n^{2*}(x))$ is a reasonable policy. Corollary 3.3 relates the three policies that I have been concerned with—the equitable policy, the policy that minimizes the maximum regret, and the policy that minimizes the total regret.

Corollary 3.3 Let z be the Pareto optimal policy related to the weighting $\lambda = 1/2$. Then:

- i) z minimizes the maximum regret,
- ii) z minimizes the total regret.

□

Again, a single policy has all three characteristics, which would make it a good starting point for any further debate.

It might be suspected that theorem 3.3 and corollary 3.3 do not depend on the assumption $j = k = 2$. Corollary 3.4 states that this is in fact so.

Corollary 3.4 If in theorem 3.3 (corollary 3.3) all the assumptions are valid except $k > 2$; $k \geq j \geq 2$, and let $z_n^{1*}(x)$, $z_n^{2*}(x)$ be respectively the minimum and maximum of the k single objective policy function, then the theorem (corollary) is true as stated.

□

Theorem 3.3 is probably more appropriate for controlling several user groups, than for allocating catch to individual boats. If allocation is being done between boats, the upper limit would not be x_t , the initial population size, but rather a series of constraints that depend on the boat capacities. However, the proof of theorem 3.3 depends only on concavity assumptions and simple upper and lower bounds. When allocating between boats there is a similar result, which modifies theorem 3.3 depending on whether the single objective optimal policy

functions are being constrained by the capacity of the boat (i.e., the boat would like to catch more but cannot) or that the optimal catch is at some other level. In this instance, there may be Pareto optimal policies where each (some) boats may catch their total capacity.

Fishery management is often based on fishing effort rather than on catch (see Beverton and Holt 1957; or Rothschild 1972 for a discussion and definitions of fishing effort). However, for many problems this distinction is more apparent than real. I consider the case where the fishing effort of the j users in period t , $e_t = (e_t^1, \dots, e_t^j)$ is normalized by what is usually called a "catchability coefficient," so that they lie in $[0, 1]$ and are additive. Moreover, assume the return to each user can be expressed as a function of the catch to each user, g^i , $k = 1, \dots, j$ and that:

- (v) each g^i satisfies assumptions (i) - (iii) as a function of the catch to user i
- (vi) total catch is a function of effort through total effort only.
- (vii) catch to each user is proportional to:

$$\frac{e^i}{\sum_{l=1}^j e^l} \cdot (\text{total catch})$$

- (viii) total catch is a continuous function of effort, nondecreasing in initial population size x for fixed effort, and nondecreasing in total effort for fixed initial population size.

Assumptions (v) - (viii) relating catch to effort satisfy most of the standard catch and effort models, such as in Anderson (1975) or Ricker (1975). To solve the effort allocation model, I show that it is

only necessary to show that any feasible catch allocation has a related feasible effort allocation. Otherwise, assumption (v) assures that the problems are the same.

Let z^1, z^2 be the allocated catch. Then it is necessary that:

$$\frac{e_1}{e_1 + e_2} (\text{total catch at } e_1, e_2) = z^1$$

$$\frac{e_2}{e_1 + e_2} (\text{total catch at } e_1, e_2) = z^2$$

Let $w^1 = e^1$, and $w^2 = e^1 + e^2$. Then by assumption (vi) total catch is a function of w^2 only, call it $TC(w^2)$. Thus given total catch, total effort is fixed also. The equations become

$$\frac{w_1}{w_2} (TC(w^2)) = z^1$$

$$1 - \frac{w_1}{w_2} (TC(w^2)) = z^2$$

$$TC(w^2), w^2 \text{ fixed; } z^1 + z^2 = TC(w^2)$$

This implies:

$$z^1 = z^2 \left(\frac{w^2}{TC(w^2)} \right) = \left(TC(w^2) - z^1 \right) \left(\frac{w^2}{TC(w^2)} \right).$$

since $z^2 + z^1 = TC(w^2)$ it implies:

$$z^1 = z^2 \left(\frac{w^2}{z^1 + z^2} \right) = z^2 \left(\frac{w^2}{z^1 + z^2} \right)$$

so that any feasible allocation (z^1, z^2) has a related, feasible allocation (w^1, w^2) (or (e^1, e^2)). Thus if assumption (v) - (viii) are valid, solving the catch allocation model solves the effort allocation model.

Other problems certainly can be approached this way. The proofs depend on concavity assumptions, simple constraints (upper and lower bounds) and the recursive formulation of the vector maximization problem: It should be no problem to analyze in a similar manner, for example, the problem where unit price is a function of total catch, so that the return is

$$p^1(\sum_1 z^1) \cdot z^1$$

where p^1 is a continuous, nonincreasing function, by using dynamic programming to first find the optimal total and then to find the optimal allocation. Other extensions are clearly desirable and I hope to follow up on them in a later paper.

IV. Discussion

The models discussed in this paper are simplifications of any true fishery. While this may be so, for many fisheries, the data does not exist to completely estimate even the models discussed here. Random variation in fish stocks is just beginning to be analyzed, and represents a significant change in our thinking, and hopefully in our policies. The results presented here should be used first and foremost to gain insight and knowledge in determining an optimal policy. It may happen, that after reflection and scrutiny, the calculated policy is used as the actual policy--but I believe this reflection and scrutiny based on our past experience and knowledge cannot be dispensed with.

The results presented here serve another purpose also. Though the exact numbers might be off, they focus the essential areas of conflict in settings relevant to the councils and the FCMA. Too often such bodies spend a lot of time on unfocused debate on points which while their objectives might differ, do not affect policy in any significant way. By pinpointing the crucial areas where policy is in conflict, and by suggesting several reasonable policies to consider at a first pass, Pareto optimal policies provide a focused starting point for council deliberations on OY.

The results here are encouraging in that finding the set of Pareto optimal policies are no more difficult than solving the single-objective optimization problems. While we cannot expect such simple results as our models for fishery management become more complex, the fact that the highly-structured nature of optimal policies is retained in the vector maximum problem, and that the known policies help determined the Pareto policies bodes well for future research in this area. In a subsequent paper, similar techniques will be used to analyze problems where harvesting gain is traded off against increased risk of undesirable events.

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APPENDIX

This Appendix contains the proofs of the various theorems. When there are two separate theorems for $k = 2$ and $k \geq 2$ they are proven together.

Some additional notation is needed. Let $V_t(\delta|H_t)$ be the (vector) expected value of following a policy δ given the history H_t up to time t . Equivalent to maximizing the weighted sum:

$$\sum_{t=1}^T \alpha^{t-1} (\sum_i \lambda^i g^i(x_t, z_t))$$

is a dynamic programming problem given by:

$$f_0^\lambda(\cdot) \equiv 0$$

$$f_n^\lambda(x) = \max \left\{ \sum_i \lambda^i g^i(x, z) + \alpha E f_{n-1}^\lambda(s[x - \sum_i z^i, d]) \right\}$$

where $f_n^\lambda(\cdot)$ denotes the dependence on a fixed value of λ .

Finally, I assume the k single objective policy functions $z_n^1(x), z_n^2(x), \dots, z_n^k(x)$ are known. For discussion purposes, I assume for fixed x , z_n^1 is the minimum of these k numbers and z_n^2 is the maximum of the k numbers. Associate with these are a dynamic programming problem denoted by

$$f_n^1(x), f_n^2(x) \text{ respectively, for each } n.$$

Theorem 2.1 Part (i). At period t , the value of the history vector H_t is a constant, call it w . Let δ be a dominated policy from period t onwards, and δ^1 an efficient policy. Then the total expected value at period 1 following the different policies is:

$$V_t(\delta^1|H_t) + w; V_t(\delta|H_t) + w$$

The difference is $V_t(\delta^1|H_t) - V_t(\delta|H_t) \geq 0$, so no nonefficient policy from period t onward can be efficient at period one.

Part (ii) is proven by a simple inductive proof, using part (i) and the independence of D_1, \dots, D_T to show that:

$$V_t(\delta|H_t) = V_t(\delta|x_t)$$

□

Corollary 2.1 Immediate consequence of theorem 2.1 and standard dynamic programming arguments.

□

Note that in corollary 2.1 the theorem is to follow some, not any, Pareto optimal policy. Therefore, in checking efficiency of some decision z , it must be tried with all possible values of f_{n-1}^{eff} before ruling it out as dominated. However, if z can be shown to be efficient at period n by some other means, then it is Pareto optimal to choose any efficient policy in period $n-1$.

Lemma A is a restatement of a well known theorem. A proof can be found in Kuhn and Tucker (1951), or Karlin (1959) or Geoffrion (1967).

Lemma A Let G map a subset of R^j into R^k , $k \geq j$, with component function g^i , $i = 1, \dots, k$. If each g^i is concave, and the constraints satisfy the Kuhn-Tucker constraint qualifications, then the following are equivalent:

(i) $z^+ \in R^j$ is a solution to $\text{vmax } G$

(ii) $z^+ \in R^j$ is a solution to:

$$\max \sum_{i=1}^k \lambda^i g^i$$

$$\text{s.t. } \sum_{i=1}^k \lambda^i = 1; 0 \leq \lambda^i \leq 1 \text{ for all } i$$

for some value of $\underline{\lambda} = (\lambda^1, \dots, \lambda^k)$.

□

For what follows, I assume that the single objective policy functions exist, and that at least one Pareto optimal policy exists. Given assumptions (i)-(iv) and the convexity of the set C , this is a very mild assumption.

Theorem 3.1 To prove the theorem, it is sufficient to prove part (i), as part (ii) follows from part (i) and theorem 2.1

Part (i) is proven for $k = 2$. For $k > 2$, assume it is true for $i = k-1$. Combine $k-1$ of the objective functions into one. This new objective function satisfies assumptions (i) - Iiii). Moreover, by induction, its optimal policy function for each x and n must lie between the largest and smallest of the $k-1$ single objective optimal policy functions. The result for $i = 2$ then proves the result for $i = k$.

It is convenient to use instead of λ , $1-\lambda$ as the weighting, to use instead 1, $\frac{1-\lambda}{\lambda}$ as the weights, where $\frac{1}{0}$ as a weight implies it is the sole objective considered. Let $u = \frac{1-\lambda}{\lambda}$ on $[0, \infty]$. Each objective obtains its highest expected return at one of the extremes. Therefore, it is sufficient to show that the optimal z is monotone and continuously varying (when it changes) as a function u . However, the monotonicity follows from lemma 7 in Geoffrion (1967) since our "overall utility" is trivially quasiconcave, V_n is concave, and the upper and lower bounds on z are constant as z varies. \square

Theorem 3.2 The equivalence of the equitable and minimum total regret policies can be seen from the equivalence of minimizing total regret to:

$$\max_{\delta} \sum_i V_n^i(\delta|x).$$

However, the equitable policy by definition is the policy which maximizes the total even-weighted expected value.

The equivalence with the Chebyshev optimal policy follows from Zangwill (1967). Zangwill proves that if the k function g^i , $i = 1, \dots, k$ are concave, then necessary and sufficient conditions for a Chebyshev optimum are:

$$\sum_{i=1}^k V^i g^i(x) \cdot w^i = 0 \quad w^i \geq 0$$

$$\sum_i w^i > 0$$

For the problem of this theorem, these are equivalent to the Kuhn-Tucker conditions for even weighting of the k objectives, which proves the equivalence of the Chebyshev optimal and equitable policies.

Theorem 3.3 This theorem is proven using a result of Karush (1959), successfully used by Veinott (1966). I assume the problem has a unique solution for each λ . Let $w^1 = z^1$, $w^2 = z^2$, ..., $w^j = \sum_{i=1}^j z^i$. Consider $J_n^\lambda(x, z)$, and substitute in for z . Then:

$$f_n^\lambda(x) = \max_{\substack{0 \leq w^1 \leq w^2 \\ w^2 \leq w^3 \leq w^4 \\ \vdots \\ w^{k-1} \leq w^k \leq x}} \left\{ \sum_{i=1}^k \lambda^i g^i(w^{i+1} - w^i) + \alpha E f_{n-1}^\lambda(s[x - w^k, d]) \right\}$$

The desired result is to show that the problem can be divided into a dynamic recursion involving only w^k , and a simple, static allocation problem given w^k . The proof is for $k = 2$. The proof $k > 2$ is by inducting on k . Assume the theorem is true at $k-1$. Combine the $k-1$ objectives into one single objective (weighted combination when appropriate). Then the result at $k = 2$ implies the result at k .

For fixed (λ, w^2) , let $A(w^2)$ be the solution to:

$$\max_{w^2 \geq w^1 \geq 0} \lambda g^2(w^2 - w^1) + (1 - \lambda) g^1(w^1)$$

Then the total return is $\lambda g^2(w^2 - A(w^2)) + (1 - \lambda) g^1(A(w^2))$ which is a function solely of w^2 . The dynamic recursion then is:

$$f_n^\lambda(x) = \max_{0 \leq w^2 \leq x} \{ \lambda g^2(w^2 - A(w^2)) + (1 - \lambda) g^1(A(w^2)) + \alpha E f_{n-1}^\lambda(s[x - w^2, d]) \}$$

which depends only on w^2 . Thus, given any w^2 in period n , the value of w^1 (ignoring the trivial case $\lambda = 0$ or 1) is either zero, w^2 or the point where:

$$\lambda g^{2'}(w^2 - w^1) = (1 - \lambda) g^{1'}(w^1)$$

which proves part (ii) of the theorem.

Again, change the weightings by allowing the first objective to be weighted by 1, and the second objective by $\frac{1-\lambda}{\lambda}$. As in theorem 3.1, it is sufficient to show that an optimal w^2 is monotone in u in order to prove the theorem.

As an induction hypothesis, assume $\frac{\partial}{\partial u} J_n^u [2](x, w^2)$ is nondecreasing for $w^2 \leq z_n^2(x)$. At period 2:

$$\frac{\partial}{\partial u} J_2^u [2](x, w^2) = g^{2'}(w^2) - \alpha E(g^{2'}(s[x - w^2, d])s^{[1]}[x - w^2, d])$$

so that the hypothesis is trivially true at period 2. This implies:

$$\frac{\partial}{\partial u} f_2^u(x) = g^{2'}(w_2^2(x))$$

unless $z_2(x)$ is at the boundary, so that since $z_2(x)$ increases with u ,

$\frac{\partial}{\partial u} f_2^u$ is decreasing in u .

At period n :

$$\frac{\partial}{\partial u} J_n^u [2](x, w^2) = g^{2'}(w^2) - \frac{\partial}{\partial u} \alpha E \left\{ f_{n-1}^u(s[x - w^2, d])s^{[1]}[x - w^2, d] \right\}$$

The second term is decreasing in u , which implies that an optimal w^2 must increase with u , which in turn implies that $\frac{\partial}{\partial u} J_n^u [2](x, w^2)$ is non-decreasing for $w^2 \leq z_n^2(x)$.

To complete the induction, unless the optimal z is x ,

$$\frac{\partial}{\partial u} f_n^u(x) = g^{2'}(w_n^2(x)) \text{ which again decreases with } u.$$

Theorem 3.4 The equivalence of the minimum total regret and the equitable policies is implied by the fact that minimizing total regret is the same as maximizing total unweighted return. Theorem 3.3 implies that the result can be put in terms of a single decision variable w^2 , and the expected return to each objective is (for $k = 2$):

$$\begin{aligned} E \sum_{t=1}^T \alpha^{t-1} g^1(A(w_t^2)) &= E \sum_{t=1}^T \alpha^{t-1} J^1(w_t^2) \\ E \sum_{t=1}^T \alpha^{t-1} g^2(w_t^2 - A(w_t^2)) &= E \sum_{t=1}^T \alpha^{t-1} J^2(w_t^2) \end{aligned}$$

Thus we are back in the one decision variable case as in theorem 3.2, which implies the equivalence of the minimize the maximum regret policy.

□

GLOSSARY AND NOTATION

- FCMA - The Fishery Conservation and Management Act of 1976
(Public Law 94-265)
- OY - optimum yield
- T - the number of period in the planning horizon
- t - subscripts the period
- n - equal to $T - t + 1$, subscripts the number of periods remaining
in the planning horizon
- x_t - stock size at the beginning of period t
- z_t^i - harvest of user i during period t
- z_t - (z_t^1, \dots, z_t^j)
- j - number of users
- k - number of total objective functions, $k \geq j$
- $g^i(x, z)$ - one period return to user i given (x, z)
- $G(x, z)$ - $(g^1(x, z), \dots, g^k(x, z))$
- D_t - independent random variable distributed as generic random
variable D
- e_t^i - fishing effort of user i
- e_t - (e_t^1, \dots, e_t^j)
- E - conditional expectation
- $s[x - \sum_{i=1}^j z_t^i, D_t]$ - population dynamics (transition function) with
catch as decision variable
- $s[x, E, D_t]$ - transition function when effort is decision
variable

- H_t - the history up to period t (a feasible sequence of states and decisions)
- $vmax$ - vector maximization (efficient set solutions)
- $f_n^{eff}(x)$ - expected vector-value of following some efficient policy from period n onwards when the state is x
- f' - derivative
- ∇f - gradient, with elements $\nabla^i f$
- $f^{[i]}$ - gradient with respect to the i^{th} vector argument
- α - discount factor; $0 < \alpha \leq 1$
- X - set of all possible states
- C - set of possible states and decisions, i.e.
- $$C = \{(x, z) : x \in X; \sum_1^i z^i \leq x, z^i \geq 0\}$$
- $z_n^{i+}(x)$ - optimal policy function in period n for objective i when it is the sole objective, $i = 1, \dots, k$
- $e_n^{i+}(x)$ - optimal effort policy function for objective i , $i = 1, \dots, k$
- $V_t(\delta|H_t)$ - the vector of expected values in period t of following policy δ given the history is A_t
- $f_n^\lambda(x)$ - total expected value of an optimal policy in period n , given state x , and a convex combination of the k objectives given by λ
- $I_n(\lambda)$ - $f_n^\lambda(x)$ as a function of λ
- $f_n^i(x)$ - total expected value of an optimal policy in period n , given state x , when objective i is the sole objective
- w^i - either $\sum_{k=1}^i z^i$ or $\sum_{k=1}^i e^i$ depending on the context.